Electronic Market-Makers: Empirical Comparison

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Abstract. Market-makers have been used by global stock exchanges as well as prediction markets to maintain liquidity and orderly price transitions in the financial markets. We use an agent-based model of the financial markets to analyze the behavior of market-makers employing various strategies. We empirically evaluate the performance of the market-makers in the financial market to demonstrate the strengths and weaknesses of the existing market-maker strategies and to test them against each other.

Keywords: Market-maker, electronic market, market-maker simulation.

1 Introduction

There has been a large increase in electronic exchanges and automatic trading in the past few years. Currently many modern exchanges, such as NYSE, NASDAQ, and Toronto Stock Exchange and even prediction markets, such as tradesports.com, use agents, called *market-makers*. In the past market-makers were dominantly special type of human brokers, whose function was to "make" the market for a particular stock. This market-maker holds a certain number of stock shares in his or her inventory with a purpose of being able to sell them to the person bidding to purchase them, or to buy stock from a seller offering stock shares. In some cases, the market-maker may just match up the buyer and seller of the stock shares. In the USA, the American Stock Exchange and the New York Stock Exchange have a single exchange member, called stock specialist, who acts as the market-maker. On the NASDAQ and other US stock and commodity exchanges, there are many competing market-makers for any given security. For example, the NASDAQ market is composed of over 300 market-makers. To ensure an efficient and effective market, there are on average 14 market-makers per stock on the NASDAQ exchange market. Each market-maker competes with the other market-makers for each stock deal that is offered. Multiple market-makers allow the market to remain competitive and fair for everyone involved.

Recently, there has been an increased interest and usage of an electronic market-makers. GETCO [7] is one of the examples of an electronic market-maker, that buys and sells securities to provide two-sided markets on exchanges around the world. Electronic market-makers use technology to create efficiencies and reduce trading costs for investors. Electronic market-makers are also responsible for maintaining the liquidity and orderly price transitions [17]. A market is called a *liquid market* if traders can buy or sell large quantities of the security at an acceptable transaction cost. Liquidity is a valuable characteristic of a market because it allows traders to realize more profits from dynamic trading. For example, a trader may want to make a transaction right away because it has some private information about the future value of the security or because it may want to optimally balance its portfolio. By fixing the problem of sparsely populated markets, market-makers allow markets to be more orderly and prices to be less volatile.

The fundamental role of the market-maker is to bring buyers together with sellers so that the traders can occur in an efficient and fair manner. Automating the market-maker functionalities can fulfill this role, making markets more productive and more stable. Market manipulation can be avoided using automated market-makers instead of human dealers and we can get more in-depth insight into the behavior of the dealers. It is important to consider the economic incentives and behaviors of electronic market-makers.

As the role of the market-makers grows, the need for better understanding of the impact of the market-makers in the market increases as well. In this paper we use the model of financial market with multiple market-makers to study the potential impact of widespread market-maker usage on market dynamics. Also, we investigate different algorithms and strategies for automated market-making in financial markets, with the goal of testing the existing strategies against each other to examine their strengths and weaknesses in a simulated environment.

2 Related Work

Automation of market-makers's functions was suggested more than three decades ago [1]. A lot of the previous research on market-making is mostly concerned with the sources and components of the bid-ask spread. A number of models have been developed to explain the evolution of the spread [9]. The problem with such approaches is that they are mostly explanatory in nature. The contributions of these models are limited to theoretical understandings of the economics of the market-making process under simplified assumptions.

Garman describes a model with a single, monopolistic market-maker, who sets prices, receives orders and clears trades and tries to maximize expected profit per unit time [6]. Such market-maker fails when it runs out of inventory or cash. In [13], the authors study the optimal behavior of a single market-maker who gets a stochastic demand. Such market-maker tries to maximize its expected utility of final wealth, which depends on the profit it receives from trading.

The information-based approach to modeling the market-makers can be a purely informational phenomenon. Glosten and Milgrom [9] investigate the market-making model with asymmetric information. Das [5] empirically studies different market-making strategies and concludes that a heuristic strategy that adds a random value to zero-profit market-makers improves the profits in the markets.

Gu [10] explores changing the market-maker behavior. His analysis includes estimating the market-maker profitability under different parameters. The results show that a profit-maximizing market-maker's objectives may not align with price variance minimization, which can be one of the qualities of an orderly market. Westerhoff [16] also explores the impact of inventory restrictions in a setup with an implied market-maker. The market-maker price adjustment reactions differ depending on the current inventory position along with current excess demands. The market-maker is assumed to make greater price adjustments when these two variables are of the same sign.

Market-making has also been adopted as a test-bed for new Machine Learning techniques [14] with a goal to demonstrate the general effectiveness of a learning algorithm, as opposed to treating market-making as a problem that requires solving. Also, empirical work has demonstrated the limitations of hard-coding market-making rules into an algorithm [11]. In [15] the primary goal is to optimally change the spread over the next iteration instead of finding the best model for past transactions.

Several market-maker strategies have been proposed and there have been a few studies on the market-maker's effect on the market. However, there does not exist a study comparing market-maker strategies in the market with multiple market-makers. Most of the past studies focus on a market with a single market-maker or a market with multiple market-makers of the same strategy. In this paper, we attempt to provide more realistic results examining the market with multiple market-makers employing different competing strategies. We also analyze the affect of each market-making strategy and the combinations of strategies on the market quality.

3 Model

We have adapted a well-known Glosten and Milgrom [9] model of financial markets used in [5] to a multi-agent framework of a financial market with multiple electronic market-makers. In our model, each human trader is modeled as a software agent, called a trading agent, that embodies the behavior of a human trader.

Market consists of N traders and M market-makers who buy and sell securities/stocks, where $M << N$. Each *trading episode* e consists of T *trading periods*. Each stock s, has a true, fundamental value $V_{s,e}$ at trading episode e. That is, there is some exogenous process that determines the value of the stock. The true price is different from the market price, which is determined by the interaction between the market-makers and the traders. $V_{s,e}$ gets updated during each trading episode with some probability $\pi_{s,e+1}$ according to the following equation:

$$V_{s,e+1} = V_{s,e} + U(\mu_{Vs,e}, \delta_{V_{s,e}}) \tag{1}$$

, where $\mu_{Vs,e}$ and $\delta_{V_{s,e}}$ are the mean and variance of the normal distribution. The jump of the true value of the stock can correspond to the news about the stock arriving to the market. The volatility of the stock value is influenced by the value of the standard deviation of the jump and the probability that the jump will occur.

The market bid(buy) price $P^{buy}_{s,t}$ at trading period t for stock s is the maximum of the market-makers' bid prices. The market ask(sell) price $P^{sell}_{s,t}$ at trading period t for stock s is the minimum of the market-makers' ask prices. We assume relatively low intelligence on the part of the traders and different strategies for market-makers that are described in Section 3.2.

The different parameters used in our prediction market model to define the market characteristics and specify the market-makers and trading agents behavior are shown in Table 3 and described below.

Market	**Parameters**
e	Trading episode
t	Trading period
N	Number of traders in the market
M	Number of market-makers in the market
S	Number of stocks
$V_{s,e}$	True value of stock s during trading episode e
$\pi s, e$	Probability that the jump in the true value of stock s occurs during trading episode e
$P^{sell}_{s,t}$	Market sell price of the stock s at trading period t
$P^{buy}_{s,t}$	Market buy price of the stock s at trading period t
Market-Maker Agent	**Parameters**
$p^{sell}_{m,s,t}$	Market-maker m's sell price of the stock s at trading period t
$p^{buy}_{m,s,t}$	Market-maker m's buy price of the stock s at trading period t
θ_m	Risk coefficient of the market-maker agent m
u_m	Market-maker m's utility

3.1 Traders

At each trading period t, traders place buy or sell order, or no order at all, based on the quote given by the market-maker. Each trader n has a valuation for each stock s, $W_{n,s} = U(V_{s,e}, \delta^2_W)$. If $W_{n,s} > P^{sell}_{s,t}$, the trader buys one unit of the stock s, if $W_{n,s} < P^{buy}_{s,t}$, the trader sells one unit of the stock s, and if $P^{buy}_{s,t} \leq W_{n,s} \leq P^{sell}_{s,t}$, the trader holds the stock.

3.2 market-makers

At each trading period t, the market-makers set bid and ask prices for each stock according to some algorithm. The difference between the bid and ask prices is called the stock's *spread*. Market-makers take on a big risk by holding a large volume of shares in their inventory, when they don't have a buyer lined up yet. They could potentially lose a large amount of money, for example if the stock drops while they have the shares in their inventory. Because of this risk, a market-maker keeps a spread on the stocks they have. The difference between ask and bid price maybe very small (pennies), but because of the large volume of shares, market-makers can still make a considerable amount of profit. Market-makers operate in the continuous market, where they execute the orders whenever they arrive. Market-maker does not know the true value of the stock, but it gets information about the news that arrive to the market about the stock, i.e. the jump in the true value of the stock.

Figure 1 shows the operations of the market-maker in a market.

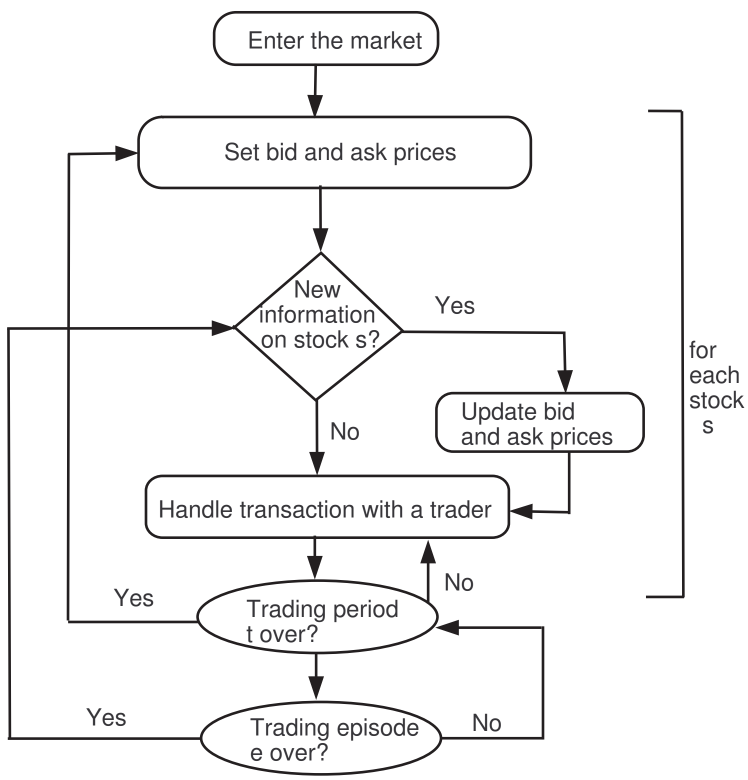

Fig. 1. A flowchart showing the operation of the market-maker agents in the market.

Before we present our experimental results, we first briefly review the market-maker algorithms that we use for our comparisons.

A Myopically Optimizing Market-Maker A Myopically Optimizing Market-Maker uses an algorithm developed by Das in [5]. The key aspect of the algorithm is that the market-maker uses the information conveyed in trades to update its beliefs about the true value of the stock, and then it sets buy/ask prices based on these beliefs. The market-maker maintains a probability density estimate over the true price of the stock.

There are two key steps involved in the market-making algorithm. The first is the computation of bid and ask prices given a probability density estimate over the true price of the stock, and the second is the updating of the density estimate given the information implied in trades. This market-maker optimizes myopically, setting the prices that give the highest expected profit at each trading period. That is, market-maker m sets buy and sell prices for security s during trading period t as follows $p^{buy}_{m,s,t} = E(V_s|Sell)$ and $p^{sell}_{m,s,t} = E(V_s|Buy)$. The market-maker uses the Bayesian updating method described in [4] to update its density estimates. All of the points in the density estimate are updated based on whether a buy order, sell order, or no order was received. The density estimate is initialized to be normal.

Reinforcement Learning Market-Maker Chan and Shelton [2] have modeled market-making problem in the framework of reinforcement learning. They have used Markov decision process (MDP) to model reinforcement learning of a market-maker. State is defined as $s_t = (inv_{m,t}, imb_t, qlt_t)$, where $inv_{m,t}$ is the market-maker m's inventory level, imb_t is the order imbalance, and qlt_t is the market quality at trading period t. Inventory level is the market-maker's current holding of the stock. The order imbalance is calculated as the sum of the buy order sizes minus the sum of the sell order sizes during a certain period of time t. Market quality measure bid-ask spread and price continuity, which refers to the amount of price change in subsequent trades. Given the states of the market, the market-maker reacts by adjusting the bis/ask prices and trading with incoming orders. The action vector for market-maker m is defined as $a_{m,t} = (p^{buy}_{m,t}, p^{sell}_{m,t})$.

The market-makers can obtain the optimal strategy by maximizing the profit, by minimizing the inventory risk, or by maximizing market qualities. Thus, the reward at each time step depends on the profit received, the change of inventory, and the market quality measures. This strategy assumes the risk-neutrality of the market-maker.

Utility Maximizing Market-Maker with Risk attributes Previous research [8, 18] has shown that by considering risk-taking and risk-averse behaviors of the human traders, the behavior of the market can be improved. We set out to see if the incorporating the risk behavior of the market-makers can improve the financial market performance. Following [8], we adopt a constant relative risk averse (CRRA) utility function $\widetilde{u}_{m,t}$ for market-maker m with a relative risk aversion coefficient. CRRA utility functions have been widely used to model risk behaviors. Relative risk aversion coefficient, θ_m, is used to classify market-maker m's risk levels as follows. If $\theta_m > 0$, the market-maker m is risk-averse, if $\theta_m = 0$, the market-maker m is risk-neutral, and if $\theta_m < 0$, the agent m is risk-seeking. Unless otherwise specified, the market-makers' risk coefficients are normally distributed in our simulations. Following the trading agent utility model in [8], during each trading period t market-maker m uses its instantaneous utility $\acute{u}_{m,t}$ and its risk-taking coefficient to calculate its modified instantaneous utility for that trading period, using Equation 2.

$$\widetilde{u}_{m,t}(\acute{u}_{m,t}, \theta_m) = \begin{cases} \frac{\acute{u}_{m,t}^{1-\theta_m}}{1-\theta_m} & \text{, if } \theta_m \neq 1; \\ ln(\acute{u}_{m,t}) & \text{, if } \theta_m = 1. \end{cases} \tag{2}$$

These market-maker agents are utility maximizers, that is they update prices so that their overall utility is maximized.

LMSR Market-Maker Hanson invented a market-maker for the use in prediction market applications called the logarithmic market scoring rule (LMSR) market-maker [12]. We have used Chen and Pennock's formulation of Hanson's (LMSR) market-maker [3]. Let $\overline{q} = (q_1, q_2 ... q_N)$ be the vector specifying quantities of stocks held by the different trading agents in the market. The total cost incurred by the trading agents for purchasing these stocks is calculated by the market-maker using a cost function $C(\overline{q}) = b \cdot ln(\sum_{j=0}^{|\overline{q}|} = e^{q_j/b})$. The parameter b is determined by the market-maker and it controls the maximum possible amount of money the market-maker can lose as well as the quantity of shares that agents can buy at or near the current price without causing massive price swings. If an agent purchases a quantity δ_q of the security, the market-maker determines the payment the agent has to make as $p_{s,m,t}^{buy} = C(\overline{q} + \delta_q) - C(\overline{q})$. Correspondingly, if the agent sells δ_q quantity of the security, it receives a payment of $p_{s,m,t}^{sell} = C(\overline{q}) - C(\overline{q - \delta_q})$ from the market-maker.

4 Experimental Results

We have compared the four market-maker algorithms described in the previous section through several simulations. The true value for stock s during episode s was obtained from the data of real NASDAQ stock markets. Each trading episode consists of 100 trading periods, where each trading period lasts for 0.5*sec*. We simulate the financial market with 100 traders and 3 or 2 market-makers.

First we want to observe the behavior of the market with pure combination, that is the market where all the market-makers use the same strategy. After that we perform the pairwise comparison of different market-maker strategies and evaluate the performance of each one in more detail. We report the market price, the spread, and the utility earned by the each type of the market-maker used in our simulations. The market price and the spread evaluate the quality of the market, whereas the utility evaluates the profitability of the strategy employed by the market-maker. In our graphs we show the results for the Yahoo! stock.

In our first set of experiments there are 3 market-makers that use the same strategy in the market. Figure 2 shows the simulations of the market with myopically optimizing market-makers. We can see from the spread graph that the myopically optimizing market-maker is sensitive to the price variations in the market. The spread value has large fluctuations following the jump in the true value of the stock. Spread seems to stabilize somewhat until the next jump. Due to large jumps in the spread value, myopically optimizing market-makers are able to keep increasing their utilities. Myopically optimizing market-makers are able to avoid causing big jumps in the market price, which is one of the important functions of the market-makers.

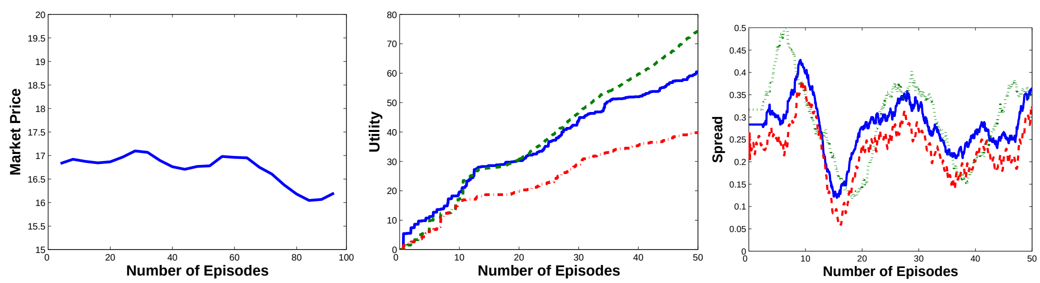

Fig. 2. Myopically Optimizing Market-Makers.

Figure 3 shows the simulations of the market with reinforcement learning market-makers. The utility of the reinforcement learning market-makers is expected to improve with each trading episode. As expected, these market-makers perform very well with respect to utility-maximization. However, the spread value fluctuates somewhat throughout the trading episodes. The market price does not fluctuate a lot throughout the simulation.

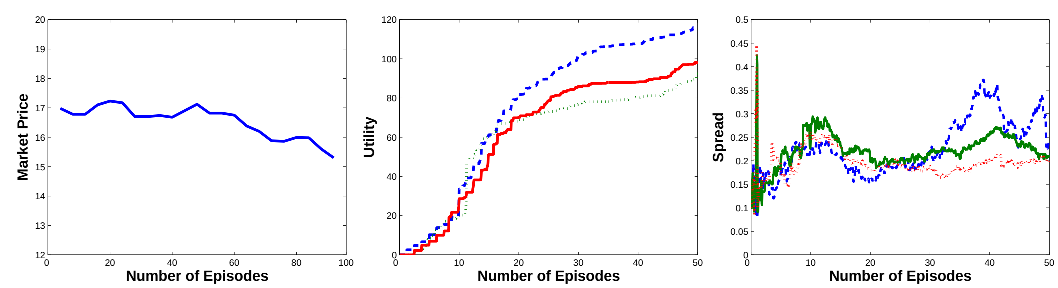

Fig. 3. Reinforcement Learning Market-Makers.

Figure 4 shows the simulations of the market with logarithmic market scoring rule (LMSR) market-makers. LMSR market-makers perform very well the function of maintaining an orderly market. That is, the market price is smooth and the spread is steady and consistent. LMSR market-makers do not aggressively maximize their utility, as can be seen from the utility graph in Figure 4.

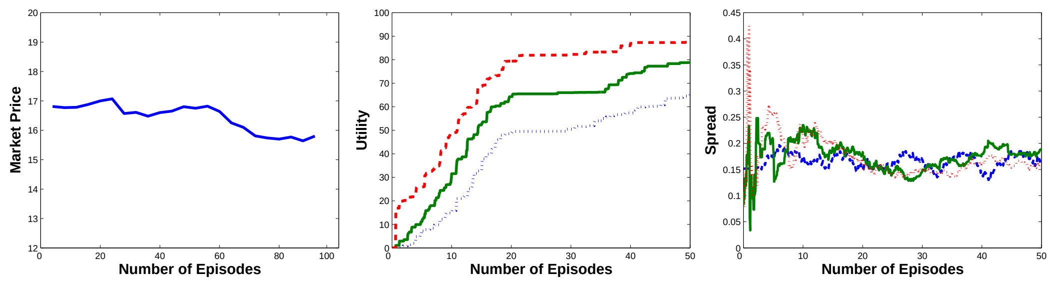

Fig. 4. LMSR Market-Makers.

Figure 5 shows the simulations of the market with utility maximizing market-makers with different risk attributes, i.e. with one risk-taking, risk-neutral, and risk-averse market-maker. We can see that the risk-taking market-maker is able to obtain slightly higher utility than the risk-neutral and risk-averse market-makers. Risk-averse market-maker gets the least utility, but maintains the smallest spread. Risk-taking market-maker does not control the spread value well, as it fluctuates a lot and by large amounts. Also, the market price has more fluctuations with these market-makers than with other types of market-makers.

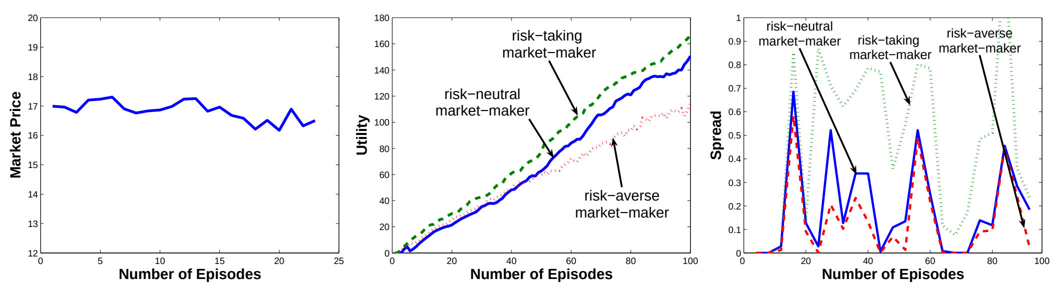

Fig. 5. Utility Maximizing Market-Makers with different risk attributes.

For our next set of simulations we compare different market-maker strategies pairwise. We simulate the market with 2 market-makers, one of each type. However, when comparing utility maximizing market-makers with 3 different risk attributes, we use 4 market-makers in the market.

First we compare myopically optimizing market-maker with 3 utility maximizing market-makers, one risk-taking, one risk-neutral, and one risk-averse market-maker. As can be seen from Figure 6, the fluctuations in the market price are pretty significant. We foresee that this is mainly due to presence of utility maximizing market-makers, since their primary function is not the control of the quality of the market, but utility maximization. Although, it is interesting to see that the risk-averse utility maximizing market-maker is able to maintain steady and low spread, and is very compatible in that regard with the myopically optimizing market-maker. Myopically optimizing market-maker also outperforms the risk-averse market-maker in overall utility.

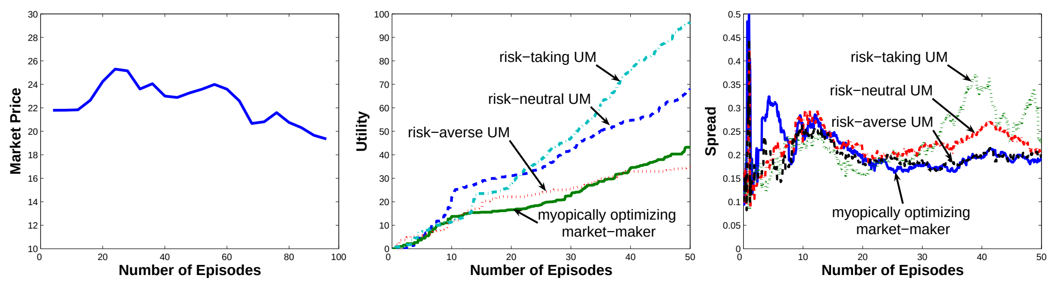

Fig. 6. Myopically Optimizing Market-Maker versus Utility Maximizing Market-Makers with different risk attributes.

Figure 7 illustrates the market with one myopically optimizing market-maker and one LMSR market-maker. We can see that these market-makers contribute to maintaining smooth market price and close spread values. However, myopically optimizing market-maker outperforms the LMSR market-maker by 40% on average in utility.

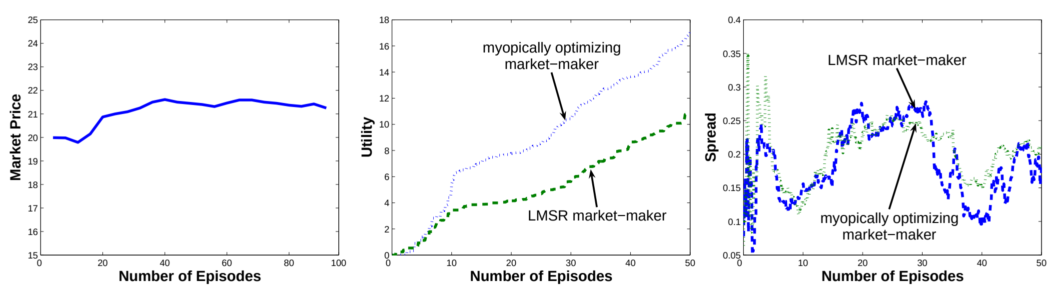

Fig. 7. Myopically Optimizing Market-Maker versus LMSR Market-Maker.

In Figure 8 we present the comparison of the myopically optimizing market-maker with reinforcement learning market-maker. Our results show that reinforcement learning market-maker is able to obtain 24% higher utility on average than the myopically optimizing market-maker. However, myopically optimizing market-maker maintain 6.5% less spread on average than the reinforcement learning market-maker. Both market-makers do a good job in maintaining smooth market price and steady spread.

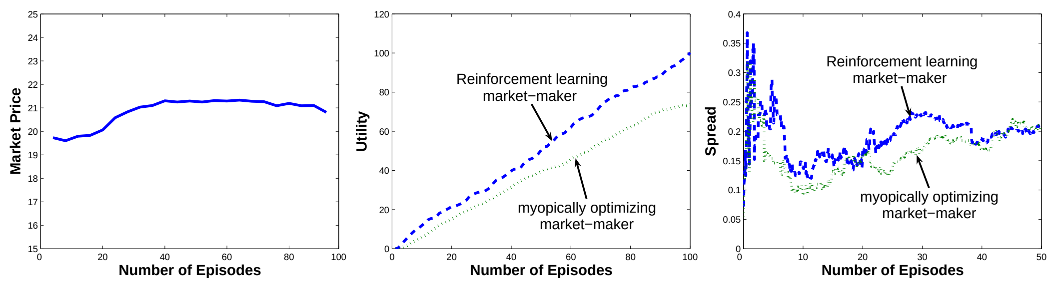

Fig. 8. Myopically Optimizing Market-Maker versus Reinforcement Learning Market-Maker.

Next we compare the performance of the LMSR market-maker with 3 utility maximizing market-makers, i.e. risk-taking, risk-neutral, and risk-averse market-maker. We can see from Figure 9 that the volatility in the market is significant, with the fluctuations in the market price and large variations in the spread values. All utility maximizing market-makers outperform LMSR market-maker in utility. For example, risk-taking utility maximizing market-maker obtains 49% higher utility than LMSR market-maker. However, LMSR market-maker has 31% lower average spread than the risk-taking market-maker, which has the highest spread.

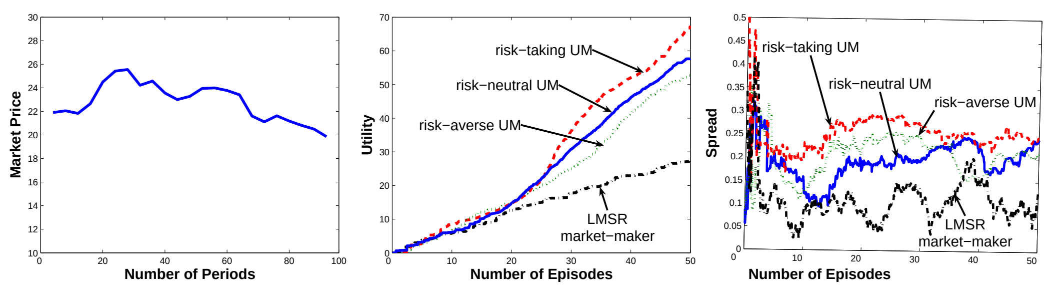

Fig. 9. LMSR Market-Maker versus Utility Maximizing Market-Makers with different risk attributes.

Figure 10 shows the performance of the reinforcement learning market-maker against the utility maximizing market-maker with different risk attributes. Our results indicate that the reinforcement learning market-maker has lower spread. In particular its average spread is 25%, 22%, and 13% lower than the risk-taking, risk-neutral, and risk-averse utility maximizing market-makers. Also, reinforcement learning market-maker is able to outperform risk-averse market-maker in utility by 11%, but it receives 69% less utility than risk-neutral market-maker, and over 100% less utility than the risk-taking market-maker.

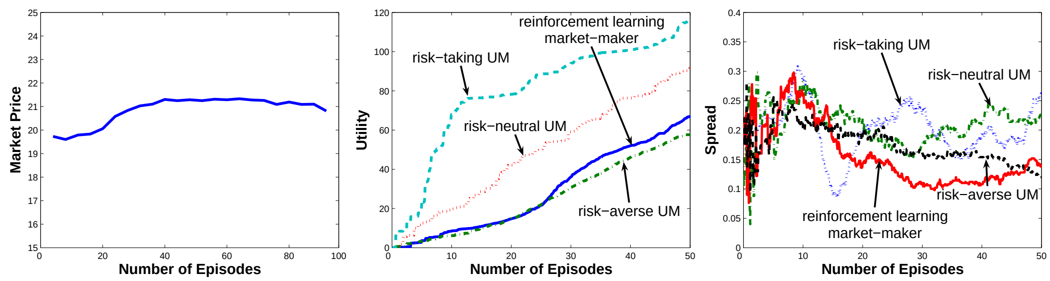

Fig. 10. Reinforcement Learning Market-Maker versus Utility Maximizing Market-Makers with different risk attributes.

Reinforcement learning market-maker performance comparison with LMSR market-maker is shown in Figure 11. The market price is smooth throughout 100 trading episodes. Although reinforcement learning market-maker obtains 54% more utility than the LMSR market-maker, the spread different between two market-makers is not very significant (8%).

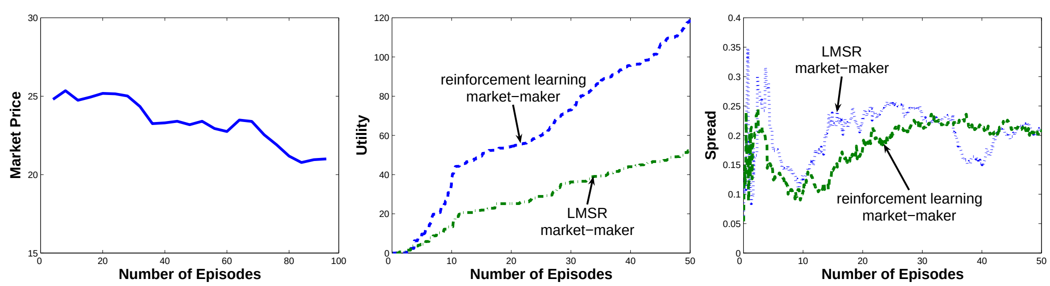

Fig. 11. Reinforcement Learning Market-Maker versus LMSR Market-Maker.

5 Conclusion

In this paper, we have used an agent-based financial market model to analyze the dynamics in the market with multiple market-makers. We investigated the effects of various market-making strategies on the market prices and market-makers' spread and utilities. The difficulty in constructing the market-making strategies comes from the need for the market-maker to balance conflicting objectives of maximizing utility and market quality, that is fine-tuning the tradeoff between utility and market quality.

Our simulation results show that the utility maximizing risk-taking and risk-neutral market-makers outperform all the other types of market-makers in utility, however they lack in maintaining the market quality, i.e. low and continuous spread and smooth market price. Myopically optimizing market-maker performs well with both maintaining good market quality and obtaining high utility. Reinforcement learning market-maker has comparable results when it comes to utility compared to the other market-maker strategies that are designed with a primary goal of maintaining market quality. Reinforcement learning market-makers also do their job of market control very well. LMSR market-maker does not do so good in terms of maximizing its utility, since it is not designed to do that. However, it performs well in maintaining continuously low spread.

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