Cooperative Patrolling via Weighted Tours: Performance Analysis and Distributed Algorithms

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Introduction

Coordinated teams of autonomous agents can effectively complete tasks requiring repetitive execution such as

1. Monitoring oil spills

2. Detecting forest fires

3. Tracking border changes

4. Surveilling an environment
Literature Survey

• In patrolling problems, it is a common approach to associate a nonnegative uncertainty value representing some measure of interest with each point in the environment.

• This uncertainty value grows with time, unless the physical location is covered by the sensor footprint of a robot.

• Additionally, a discrete representation of the environment is typically obtained by selecting a set of important viewpoints, and by creating a robotic roadmap with the viewpoints as vertices.

• The robots are constrained to move along this roadmap, and the performance of the team is measured according to a frequency of visit criteria which reflects the amount of uncertainty in the environment.


Literature Survey

• Two classes of patrolling strategies, based, respectively, on space decomposition and traveling salesman tour computation, are presented and qualitatively compared


• An efficient and distributed solution to the perimeter patrolling problem is proposed


• The computational complexity of the patrolling problem is studied in relation to the roadmap representing the environment

This paper focuses on the problem of patrolling an environment with a team of autonomous agents.

Given a set of strategically important locations (viewpoints) with different priorities, the patrolling strategy consists of:

1. Constructing a tour through the viewpoints

1. Driving the robots along the tour in a coordinated way
Contributions

• Paper consider the design of both optimal trajectories and distributed control laws for the robots to converge to optimal trajectories

1. First, we propose a patrolling strategy and we characterize its performance as a function of the environment and the viewpoints priorities

2. Second, we restrict our attention to the problem of patrolling a nonintersecting tour, and we describe a team trajectory with minimum weighted refresh time

3. Third, for the tour patrolling problem and for two distinct communication scenarios, namely the Passing and the Neighbor-Broadcast communication models, author develop distributed algorithms to steer the robots toward a minimum weighted refresh time team trajectory

4. Finally, we show the effectiveness and robustness of our control algorithms via simulations and experiments
Step and Approach

• The uncertainty growth of the viewpoints is linear, with a possibly different rate (priority) for each viewpoint

• The uncertainty value becomes zero as soon as the viewpoint is covered by the sensor footprint of a robot

• For a given set of connected viewpoints with priorities, paper first construct a minimum spanning tree through the viewpoints

• Then, by means of standard graph theory techniques, paper compute a tour through the viewpoints

• Finally, paper constrain the robots on the tour and compute an optimal team trajectory

• For optimality criteria, paper consider the weighted refresh time, which, loosely speaking, is the longest time interval between any two visits of a viewpoint, weighted by the corresponding viewpoint’s priority
Problem Setup and Preliminary Results

Robotic Model and Preliminary Concepts

• A given team of \( m > 0 \) identical robots, which are capable of sensing, communicating, and moving in a path-connected environment \( \mathcal{E} \subseteq \mathbb{R}^2 \)

• Regarding sensing, paper assume that the environment can be completely covered by simultaneously placing a robot at each of \( n > m \) viewpoints in the configuration space

• Since \( n > m \), at least one robot must visit multiple viewpoints for the entire environment to be monitored over time

• the positive priority \( \phi_\alpha \in \mathbb{R}_{>0} \) with the \( \alpha \)th viewpoint

\[
\Phi = \{\phi_1, \ldots, \phi_n\} \\
\phi_{\text{min}} = \min\{\phi: \phi \in \Phi\} \\
\phi_{\text{max}} = \max\{\phi: \phi \in \Phi\}
\]
Problem Setup and Preliminary Results

Robotic Model and Preliminary Concepts

- Robots are holonomic, i.e., modeled as first-order integrators, and move at most at unit speed

- Constrain the motion of the robots to the robotic roadmap $G = (V, E)$, where $V = \{v_1, \ldots, v_n\}$ denotes the set of viewpoints and the undirected edge $(v_\alpha, v_\beta) \in E$ denotes the possibility for a robot to travel between $v_\alpha$ and $v_\beta$

- A team trajectory $X$ is an array of $m$ continuous and piecewise differentiable trajectories $x_1(t), \ldots, x_m(t)$ defined by the motion of the robots on the roadmap $G$ i.e., $x_i: [0, \infty) \rightarrow G$, for $i \in \{1, \ldots, m\}$

- Viewpoint $v_\alpha$ is visited at time $t$ by robot $i$ if $x_i(t) = v_\alpha$
Problem Setup and Preliminary Results

Robotic Model and Preliminary Concepts

Let $A(\alpha, i)$ and $D(\alpha, i)$ be, respectively, the set of times at which robot $r_i$ arrives at and departs from the viewpoint $v_\alpha$ for a sufficiently small $\varepsilon \in \mathbb{R}_{>0}$.

$$A(\alpha, i) = \{ t \in [0, \infty) : x(t) = v_\alpha \text{ and } x(t - \varepsilon) \neq v_\alpha \}$$

$$D(\alpha, i) = \{ t \in [0, \infty) : x(t) = v_\alpha \text{ and } x(t + \varepsilon) \neq v_\alpha \}.$$
Problem Setup and Preliminary Results

Robotic Model and Preliminary Concepts

- The (weighted) refresh time of a team trajectory $X$, which we denote by $RT(X)$, is the longest weighted time interval between any two consecutive visits of any viewpoint, i.e.,

$$RT(X) = \max \left\{ \phi_\alpha \left( t_a(\alpha, t_d) - t_d \right) : \alpha \in \{1, \ldots, n\}, \ t_d \in D(\alpha, i) \text{ for any } r_i \right\}$$

$$t_a(\alpha, t_d) = \min \{ t \in \cup_{i=1}^m A(\alpha, i) : t \geq t_d \}.$$

- $t_a(\alpha, t_d)$ is the earliest arrival time by any robot at node $\alpha$ after departure at time $t_d$

- $RT(X)$ can be undefined if no viewpoint is visited by the robots

- The situation is out of interest
Problem Setup and Preliminary Results

Robotic Model and Preliminary Concepts

Problem 1: Cooperative Patrolling

Given a set of viewpoints with priorities, and a team of $m \geq 2$ robots, design a persistent team trajectory with minimum refresh time.

- The cooperative patrolling problem is, in general, computationally hard, even if all the priorities have the same value [9]
- The authors are not proposing an optimal solution to the Cooperative Patrolling Problem
- Instead, they are describing an efficient and distributed patrolling strategy with performance guarantee

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Cyclic Patrolling Strategy and Optimality Bound

Patrolling strategy consists of three steps

- First, construct a minimum spanning tree of the roadmap $G$ [13].
- Second, construct a nonintersecting tour visiting the viewpoints by doubling the edges of the computed minimum spanning tree
- Third, let the robots continuously travel the tour in a coordinated way

In particular, for a tour of length $L$, a team of $m$ robots, and a set of initial positions, define the Equal-Spacing trajectory to be such that

1. The robots continuously travel the tour at maximum speed in the same direction
2. The distance between any two consecutive robots is $L/m$
3. Notice that the refresh time of the Equal-Spacing trajectory equals $(\varphi_{max}L)/m$

Cyclic Patrolling Strategy and Optimality Bound

**Theorem**: Let $G$ be a robotic roadmap on the viewpoints $V$ with priorities $\Phi$. Let $X(t)$ be the team trajectory generated by the weighted-cyclic strategy on $G$. Then

$$\frac{RT(X)}{RT^*} \leq 2(1 + \gamma) \frac{\phi_{\text{max}}}{\phi_{\text{min}}}$$

where $RT^*$ denotes the minimum refresh time on $G$, and $\gamma$ denotes the ratio of the longest to the shortest edge of $G$.

- Observe that, if $\gamma$ and $\phi_{\text{max}}/\phi_{\text{min}}$ are bounded by constants, then the weighted-cyclic strategy is a constant factor approximation to the Cooperative Patrolling problem (Problem 1).
Patrolling of a Weighted Tour

Problem 2 (Cooperative Tour Patrolling): Given a set of viewpoints with priorities on a nonintersecting tour $\Gamma$, and a team of $m \geq 2$ robots moving on $\Gamma$ in uniform direction, design a persistent team trajectory with minimum refresh time.

Remark 2 (Single-Robot Tour Patrolling): If $m = 1$, then a solution to the Cooperative Tour Patrolling problem (Problem 1) consists of letting the robot move at maximum speed along the tour. The refresh time of such a trajectory is $\phi_{\text{max}} L$. 
**Definition 1 (Stop-Go Team Trajectory):** A team trajectory is said to be *Stop-Go* if it is order-invariant and, for each robot $r_i$, it holds $\dot{x}_i(t) \in \{0, 1\}$ with $x_i(t) \in V$ whenever $\dot{x}_i(t) = 0$.

**Lemma 2.2 (Optimality of Stop-Go Team Trajectories):** For any team trajectory $X$ on a nonintersecting tour, there exists a Stop-Go team trajectory $\tilde{X}$, satisfying $\text{RT}(\tilde{X}) \leq \text{RT}(X)$. 
Team Trajectories With Minimum Refresh Time

Equal time spacing Trajectory

**Trajectory 1: Equal-Time-Spacing Trajectory**

<table>
<thead>
<tr>
<th>Input</th>
<th>Viewpoints $V$, Priorities $\Phi_{\text{max}}$ and $\phi_1$, Tour length $L$;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Require</td>
<td>$\phi_1 = \max{\phi : \phi \in \Phi_{\text{min}}}$;</td>
</tr>
<tr>
<td>Output</td>
<td>A Stop-Go team trajectory as specified by $\delta^i_\alpha(k)$, $x_i(0)$;</td>
</tr>
</tbody>
</table>

1. $RT^*_T := \frac{L}{\sum_{\phi \in \Phi_{\text{max}}} \phi - 1}$;

   for each robot $r_i$ and iteration $k$ do
   
   for each $v_\alpha \in V_{\text{min}}$ do
   
   2. $\delta^i_\alpha(k) := 0$;
   
   for each $v_\alpha \in V_{\text{max}}$ do
   
   3. $\delta^i_\alpha(k) := \frac{\phi_\alpha - \phi_1}{\phi_\alpha \phi_1} RT^*_T$;

4. $x_1(0) := v_1$;

5. for each robot $r_i$ do

6. $x_i(0) := x_1((m + 1 - i) RT^*_T / \phi_1)$;
Theorem 3.1 (Optimality of Equal-Time-Spacing Trajectories): Given a set of viewpoints with priorities on a nonintersecting tour of length $L$, and a team of $m \geq 2$ robots, let $\Phi_{\text{max}}$ denote the set of $m$ largest priorities. Then, we have the following.

1) The Equal-Time-Spacing trajectory $X(t)$ in Trajectory 1 has minimum refresh time.

2) $RT(X) = RT_T^* = \frac{L}{\sum_{\phi \in \Phi_{\text{max}}} \phi^{-1}}$. 
Team Trajectories With Minimum Refresh Time

Example 1 (Equal-Time-Spacing Trajectory): Consider a tour of five viewpoints with priorities \( \{\phi_1, \ldots, \phi_5\} \), and a team of three robots. Assume that \( \phi_5 \leq \phi_1 < \phi_4 < \phi_3 < \phi_2 \). The Equal-Time-Spacing trajectory for this configuration is given in Fig. 3. Notice that the robots do not stop at \( v_1 \) and \( v_5 \), while they stop for a certain time interval at \( v_2, v_3, \) and \( v_4 \).

Fig. 2. Robots \( \{r_1, r_2, r_3\} \) patrol the nonintersecting tour \( \Gamma \) with viewpoints \( \{v_1, \ldots, v_5\} \). Robots move in counterclockwise direction.
Distributed Control Algorithms

We consider two different communication models:

1. The *Passing communication* model, which assumes that two robots communicate only when they occupy the same position.

2. The *Neighbor-Broadcast communication* model, where sporadically robots $i + 1$ and $i - 1$ synchronously send their position on the shared trajectory to robot $i$. 
Distributed Control Algorithms

Passing communication model

Algorithm 2: Leader-Based Control Law (leader robot)

Input : Viewpoint $v_{\text{max}}$, Priorities $\Phi_{\text{max}}$ and $\phi_1$, Tour length $L$;
Require : $\phi_1 = \max\{\phi : \phi \in \Phi_{\text{min}}\}$, $t := \text{current time}$;

1. $T^* := \frac{L}{\phi_1 \sum_{\phi \in \Phi_{\text{max}} \setminus \{\phi_{\text{max}}\}} \phi^{-1}}$;

while true do

2. $T_{\text{last}} := \text{time of last departure of a robot from viewpoint } v_{\text{max}}$;
   if robot arrives at viewpoint $v_{\text{max}}$ then
   stop robot at viewpoint $v_{\text{max}}$ for $\max\{0, T^* - (t - T_{\text{last}})\}$;

end while
Algorithm 3: Leader-Based Control Law (i-th robot)

- **Input**: Viewpoint \( \nu_{\text{max}} \), Priorities \( \Phi_{\text{max}} \) and \( \phi_1 \), Tour length \( L \);
- **Require**: \( \phi_1 = \max\{\phi : \phi \in \Phi_{\text{min}}\} \), \( t := \) current time;

1. compute the Equal-Time-Spacing trajectory with \( |\Phi_{\text{max}}| - 1 \) robots and priority set \( \Phi_{\text{max}} \setminus \{\phi_{\text{max}}\} \);
2. follow the Equal-Time-Spacing trajectory unless differently instructed by the leader when passing viewpoint \( \nu_{\text{max}} \);
Distributed Control Algorithms

Passing communication model

Lemma 4.1 (Leader-Based Control Performance): Given a set of viewpoints with priorities on a nonintersecting tour of length $L$, and a team of $m \geq 2$ robots with Passing communication model, let $\Phi_{\text{max}}$ denote the set of $m$ largest priorities. Let $X(t)$ be the team trajectory generated by the Leader-Based control law. Then

$$RT(X(t \geq t_0)) = RT^*_\ell = \frac{L}{\sum_{\phi \in \Phi_{\text{max}} \setminus \{\phi_{\text{max}}\}} \phi^{-1}}$$

i.e., the team trajectory $X(t)$ restricted to the interval $[t_0, \infty)$ has refresh time $RT^*_\ell$. Moreover

$$t_0 \leq 2L + 2 \sum_{\alpha=1}^{n} \delta^1_{\alpha}(1)$$

where, for $i = 1, \ldots, n$, $\delta^1_{i}(1)$ is defined in Trajectory 1.
Distributed Control Algorithms

Neighbor-Broadcast Communication Model

• Let

\[ \tau_i(t) = \max \{ \tau : \tau \leq t, x_i(\tau) = u_1 \} \]

• Assumption, upon communication, robot \( i \) knows the value \( \tau_{i-1}(t) \) and \( \tau_{i+1}(t) \)

• Each robot moves according to the Equal-Time-Spacing trajectory, except when otherwise specified by the control law, that robot \( i \) does not initiate any additional communication until its control action is terminated
Distributed Control Algorithms

Neighbor-Broadcast Communication Model

Algorithm 4: Stop-When-Ahead Control Law (i-th robot)

Input : Viewpoints $V$; Priorities $\Phi_{\text{max}}$ and $\phi_1$; Tour length $L$;
        Gain $q \in (0, 1/2)$; Period $T = L + \sum_{\alpha=1}^{n} \delta_\alpha (1)$;
Requirement : $\phi_1 = \max \{ \phi : \phi \in \Phi_{\text{min}} \}$; $t := \text{current time}$;

while true do

1. set $\tau_i(t) = \max \{ \tau : \tau \leq t, x_i(\tau) = v_1 \}$;
2. if communication occurs then
3. $\tau^c_i := \text{mod}(\tau_{i+1}(t) - \tau_i(t), T) - \text{mod}(\tau_i(t) - \tau_{i-1}(t), T)$;
4. robot $i$ stops for $\max \{0, q\tau^c_i\}$;
5. set $\tau_i(t) := \tau_i(t) + \max \{0, q\tau^c_i\}$;

end
Lemma 4.2 (Stop-When-Ahead Control Performance): Given a set of viewpoints with priorities on a nonintersecting tour of length $L$, and a team of $m \geq 2$ robots with Neighbor-Broadcast communication model, let $\Phi_{\text{max}}$ denote the set of $m$ largest priorities. Let each robot implement the Stop-When-Ahead control law, and let $X(t)$ be the resulting team trajectory. Assume the existence of a duration $\rho$ such that each robot communicates within the interval $[t, t + \rho]$, for each time $t$. Then

$$\lim_{t_0 \to \infty} \frac{RT(X(t \geq t_0))}{RT_T} = \frac{L}{\sum_{\phi \in \Phi_{\text{max}}} \phi^{-1}}$$

i.e., the team trajectory $X(t)$ asymptotically converges to a minimum refresh time team trajectory.
Simulation and Experiment Results

Simulation and experiment Setup

Robot Hardware

- Paper use Erratic mobile robots from Videre Design
- The vehicle platform has a roughly square footprint (40 cm × 37 cm), with two differential drive wheels and a single rear caster
- Each robot carries an on-board computer with a 1.8 GHz Core 2 Duo processor, 1 GB of memory, and 802.11g wireless communication
- For navigation and localization, each robot is equipped with a Hokuyo URG-04LX laser rangefinder
Similation and Experiment Results

*Simulation and Tour Patrolling*

- Paper simulate the Stop-When-Ahead control algorithm in a campus environment.
- The purpose of this simulation is twofold:
  1. To illustrate the effectiveness of the control law in coordinating the robots toward the desired trajectory.
  2. To show that the control law is robust to modeling uncertainties and the dimension of the problem.
Simulation and Experiment Results

Simulation and Tour Patrolling

- The environment used for this is a part of UCSB campus
- 35 points and 5 robots
- \{1, 1.04, 1.09, 1.19, 1.31\},
Similation and Experiment Results

*Simulation and Tour Patrolling*

Fig. 5. For the environment in Fig. 1, this plot shows the trajectories of five robots as obtained from the Stop-When-Ahead control algorithm. The control algorithm becomes active after the robots have completed one lap, and it steers the robots toward an Equal-Time-Spacing trajectory. Notice that, as specified in Trajectory 1, the waiting intervals at some viewpoints are nonzero.
Simulation and Experiment Results

Experiment of Tour Patrolling

Fig. 7. Indoor environment with three robots and a tour with five viewpoints.
Similation and Experiment Results

Experiment of Tour Patrolling

Fig. 8. For the environment in Fig. 8, this figure depicts the trajectories of three robots as obtained from the Stop-When-Ahead control algorithm. The control algorithm starts at time 250 and steers the robots towards an Equal-Time-Spacing trajectory. Because of motion and localization uncertainty and of the presence of obstacles, the robots do not always move at maximum speed.
Conclusions

• The problem of patrolling a set of weighted viewpoints to minimize their weighted refresh time has been considered.

• The proposed approach consists of:

  1. creating a tour through the viewpoints by means of graph-theoretic techniques

  2. instructing the robots to travel according to an Equal-Time-Spacing trajectory.